FRQ 3 Practice

Your homework is on the following pages, but here is some helpful information relevant to this type of question.

General Notes

- This question always involves modelling a periodic context.
- A figure is often, but not always, given to help illustrate the context.
- A strategy for reading the problem:
  - Step 1: Read the first line, then skip to the last line to see what $h(t)$ is measuring
  - Step 2: Read the problem looking for values of the thing $h(t)$ is measuring. Look for the maximum, minimum, midline, and amplitude of the thing $h(t)$ is measuring
  - Step 3: Read the problem again, this time looking for details about time. There will be one time given directly tied to a value of $h(t)$. Something like "At $t = 5$, the glass is completely full." And there will be enough given to find period/frequency.
- It is easy to mix up period and frequency. If 10 laps are made in 3 seconds, is the period $\frac{\text{10 laps}}{\text{3 seconds}}$? Or is it $\frac{\text{3 seconds}}{\text{10 laps}}$? Period will always be the one with time in the numerator.
- FRQ 3 is completed without a calculator.

Part A

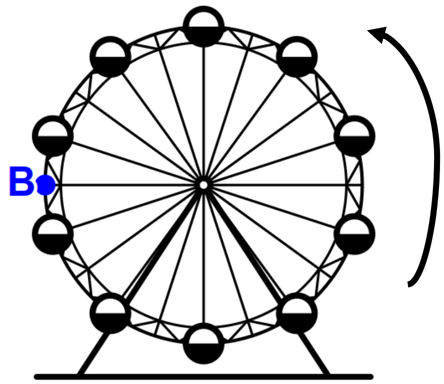
- Asks you to find the coordinates of five points. The wording for part A will never change.
- Two periods of the function will always be given in the picture with the same five points always labelled.
- The maximum, midline, and minimum will always be shown.
- The response booklet will have an image of the graph and spaces to write the coordinates.
- Negative t-values are acceptable unless something in the context of the problem clearly restricts that possibility (e.g. "The ball starts rolling at $t = 0$," "the washing machine is turned on at $t = 0$," etc)
- Fractions do not need to be simplified.
- Exact decimals are allowed. Or fractions. You do you.
- No work needs to be shown for this part on the AP exam.
- 1 point for correct input values (there are an infinite number of correct answers here)
- 1 point for correct output values (there is only one right answer)

Part B

- The wording for part B will always be exactly the same.
- It might be sine or might be cosine.
- Fractions do not need to be simplified.
- Exact decimals are allowed. Or fractions. You do you.
- No work needs to be shown for this part on the AP exam.
- 1 point for a and d.
- 1 point for b and c.

Part C

- Part (i) is not about concavity.
- Part (ii) asks two questions. Both are about concavity, just using different words.
- One can answer C(ii) without reading a single word of context.
- Say "The rate of change of $h(t)$ is decreasing on the interval (t_1, t_2)" not "The rate of change of the interval (t_1, t_2) is decreasing."

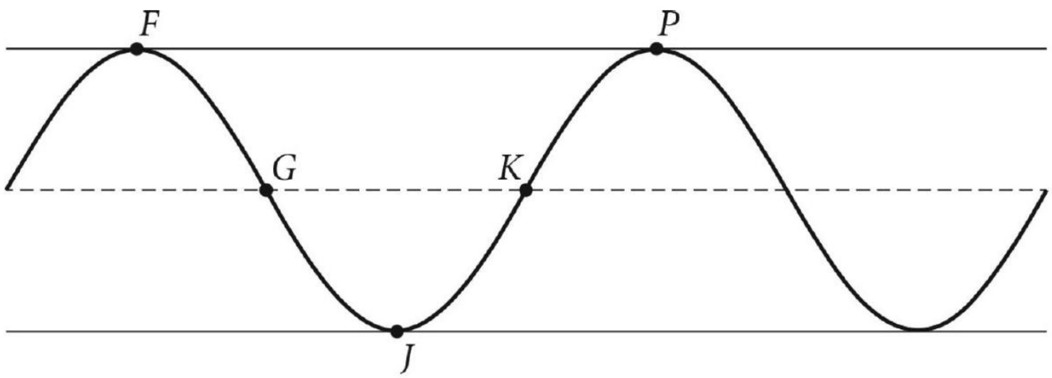


Note: Figure not drawn to scale.

1. The London Eye Ferris Wheel rotates in a counterclockwise direction and completes one revolution in twenty-eight minutes. Point B is on the edge of the Ferris wheel and is located directly to the left of the center of the Ferris wheel at time $t = 0$, as indicated in the figure. Point B is 62 meters from the center of the Ferris wheel. The center of the London Eye is 75 meters off the ground. As the London Eye rotates at a constant speed, the distance between B and the ground periodically decreases and increases.

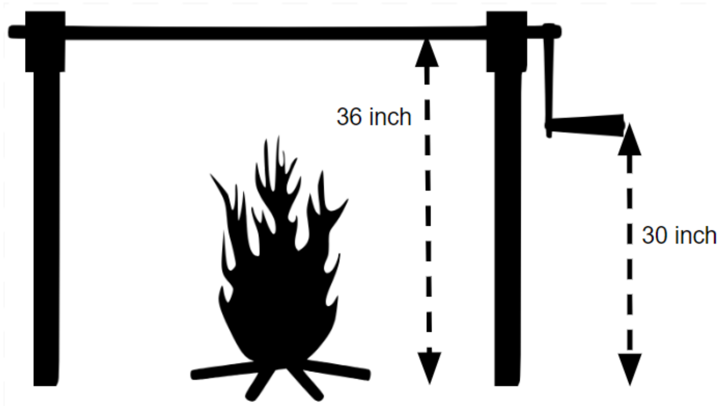
   The sinusoidal function h models the distance between B and the ground, in meters, as a function of time t in minutes.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $\left(t, h(t)\right)$ for the five points: $F, G, J, K,$ and P.



   (B) The function h can be written in the form $h(t) = a\cos\left(b(t + c)\right) + d$. Find values of constants $a, b, c,$ and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of J is t_1 and the t-coordinate of K is t_2.
   (i) On the interval (t_1, t_2), which of the following is true about h?
      a. h is positive and increasing
      b. h is positive and decreasing
      c. h is negative and increasing
      d. h is negative and decreasing
   (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

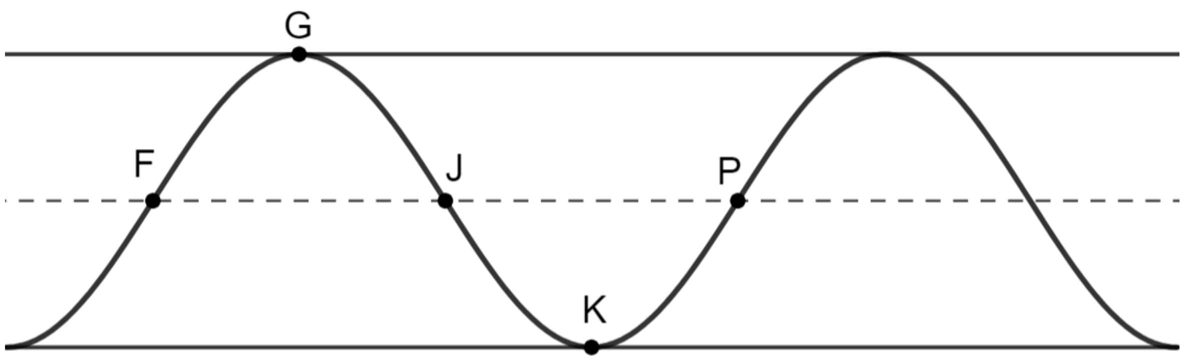


Note: Figure not drawn to scale.

2. A roasting spit is used to slowly turn food over a fire, letting it roast evenly. The spit is 36 inches off the ground, as is shown in the diagram. As the handle is cranked, the roasting spit turns and the height of the handle periodically increases and decreases. At time $t = 0$, the handle is 30 inches from the ground, the lowest height it ever reaches. The handle turns at a steady rate and makes 6 complete turns every half of an hour.

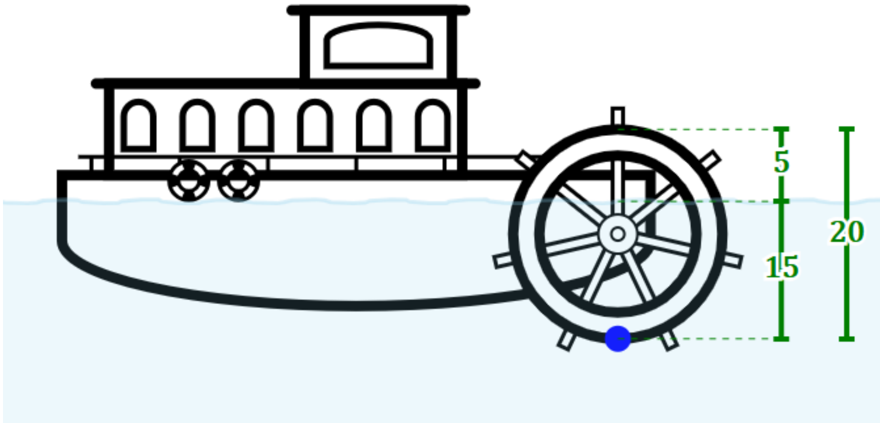
   The sinusoidal function h models the distance between the handle and the ground, in inches, as a function of time t in minutes.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $\big(t, h(t)\big)$ for the five points: F, G, J, K, and P.



   (B) The function h can be written in the form $h(t) = a\cos\big(b(t + c)\big) + d$. Find values of constants a, b, c, and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of F is t_1 and the t-coordinate of G is t_2.
   (i) On the interval (t_1, t_2), which of the following is true about h?
      a. h is positive and increasing
      b. h is positive and decreasing
      c. h is negative and increasing
      d. h is negative and decreasing
   (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

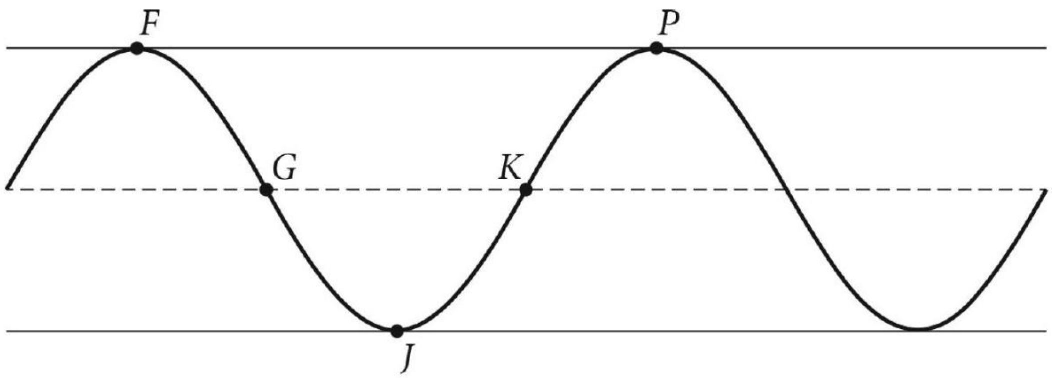


Note: Figure not drawn to scale. Also not realistic. This is some Micky Mouse nonsense.

3. On a paddle steamer ship, steam turns a circular paddle that propels the ship forward in water. On the *PS Waverly*, the paddle is a circle with diameter 20 feet. The highest a point on the paddle reaches as it turns is a height of 5 ft above the water. The lowest a point on the paddle reaches is a height of 15 ft below the water. Let this be considered a height of -15 ft. The paddle turns at a constant rate, making one complete revolution every 4 seconds. At time $t = 0$, a certain point on the paddle is at the bottom of its rotation at a height of -15 ft.

   The sinusoidal function h models the height of this point above the water, in feet, as a function of time t in seconds. h uses the convention that distance below the water is considered negative height.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $\big(t, h(t)\big)$ for the five points: F, G, J, K, and P.



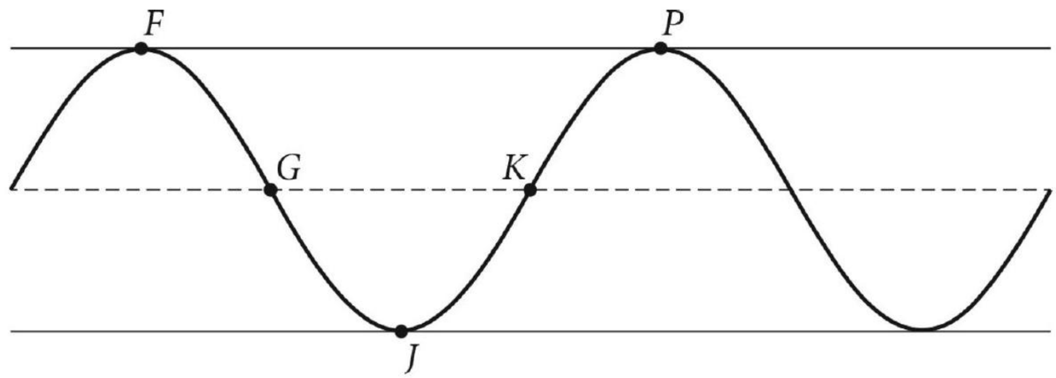
   (B) The function h can be written in the form $h(t) = a\cos\big(b(t + c)\big) + d$. Find values of constants a, b, c, and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of G is t_1 and the t-coordinate of J is t_2.
   (i) On the interval (t_1, t_2), which of the following is true about h?
   a. h is positive and increasing
   b. h is positive and decreasing
   c. h is negative and increasing
   d. h is negative and decreasing
   (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

4. In Fairbanks Alaska, days in the summer tend to have many more hours of sunlight than days in the winter. In general, the winter solstice in late December tends to have only four hours of sunlight, the shortest length of sunlight Fairbanks gets all year. Six months later, Fairbanks may get twenty-two hours of sunlight in late June during the summer solstice, the day marked by the most sunlight. From the winter solstice to the summer solstice, the amount of sunlight increases. From the summer solstice to the next winter solstice, the amount of sunlight decreases.

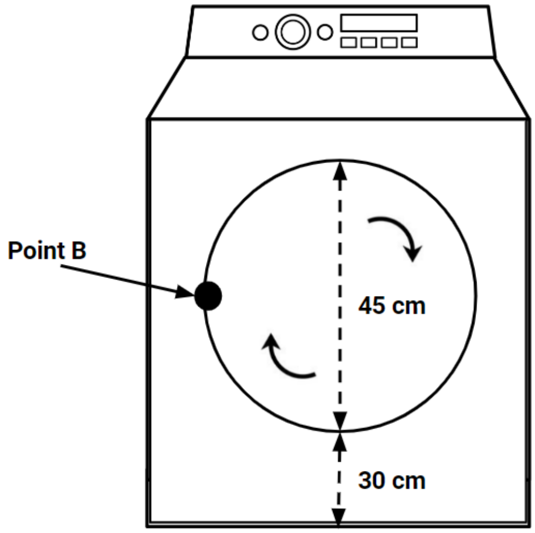
   The sinusoidal function h models the amount of sunlight, in hours, Fairbanks gets in a day as a function of time t in months since the summer solstice.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $(t, h(t))$ for the five points: $F, G, J, K,$ and P.



   (B) The function d can be written in the form $h(t) = a\sin\big(b(t+c)\big) + d$. Find values of constants $a, b, c,$ and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of G is t_1 and the t-coordinate of J is t_2.
      (i) On the interval (t_1, t_2), which of the following is true about h?
         a. h is positive and increasing
         b. h is positive and decreasing
         c. h is negative and increasing
         d. h is negative and decreasing
      (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

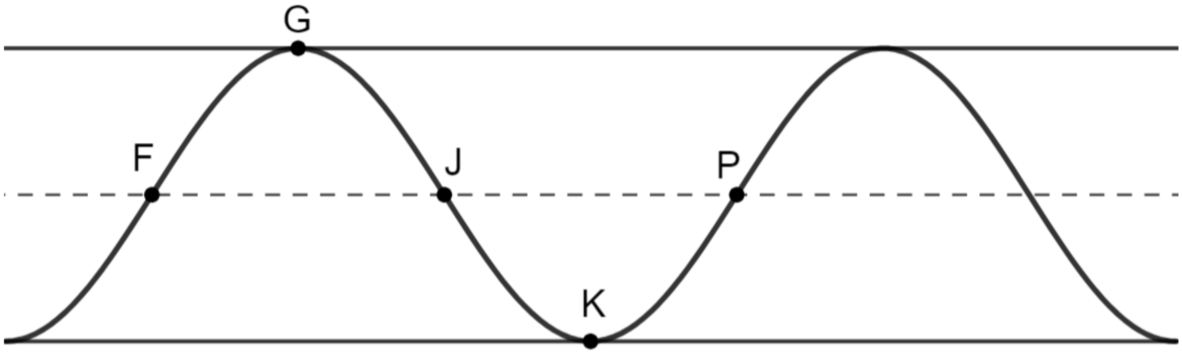


Note: Figure not drawn to scale.

5. Front loading washing machines spin clothes using a cylindrical drum. On a certain washing machine, the drum has diameter 45 cm. Point B is a specific point on the edge of the drum. During the machine's spin cycle, Point B makes 1200 revolutions every minute, spinning at a steady rate in the clockwise direction. The closest to the ground point B ever reaches is 30 cm from the ground. Let time $t = 0$ refer to a time at which point B is directly to the left of the center of rotation, as shown in the diagram.

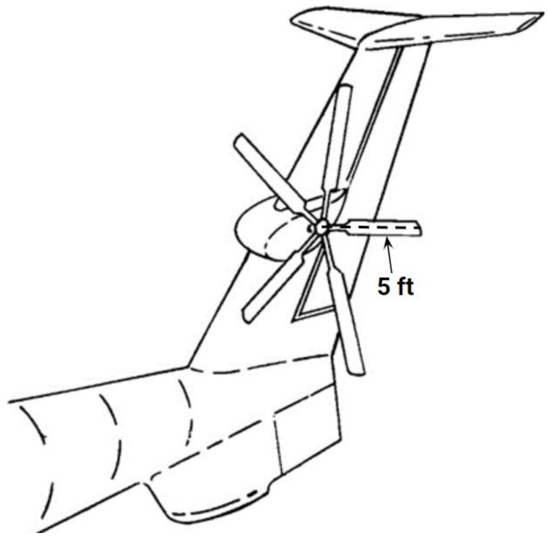
   The sinusoidal function h models the distance between B and the ground, in centimeters, as a function of time t in seconds.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



   (B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of G is t_1 and the t-coordinate of J is t_2.
   (i) On the interval (t_1, t_2), which of the following is true about h?
      a. h is positive and increasing
      b. h is positive and decreasing
      c. h is negative and increasing
      d. h is negative and decreasing
   (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

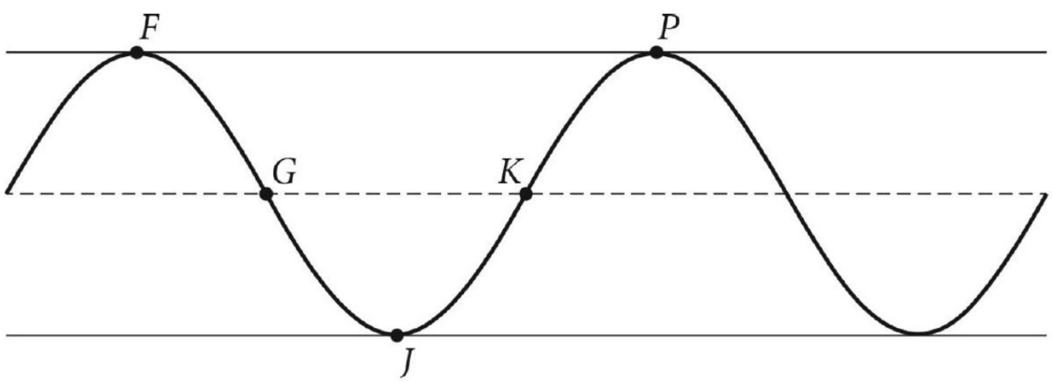


Note: Figure not drawn to scale.

6. The tail rotor on a Sikorsky S-72 RSRA helicopter consists of five blades, and each blade is 5 feet long. During maintenance, the rotor is spun at a constant rate of 5 rotations per minute, and the center of the rotor is 12 ft above the ground. At time $t = 0$, a point on the tip of one of the rotor's blades is at its maximum height.

   The sinusoidal function h models the distance between that point and the ground, in feet, during maintenance as a function of time t in seconds.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $\left(t, h(t)\right)$ for the five points: $F, G, J, K,$ and P.



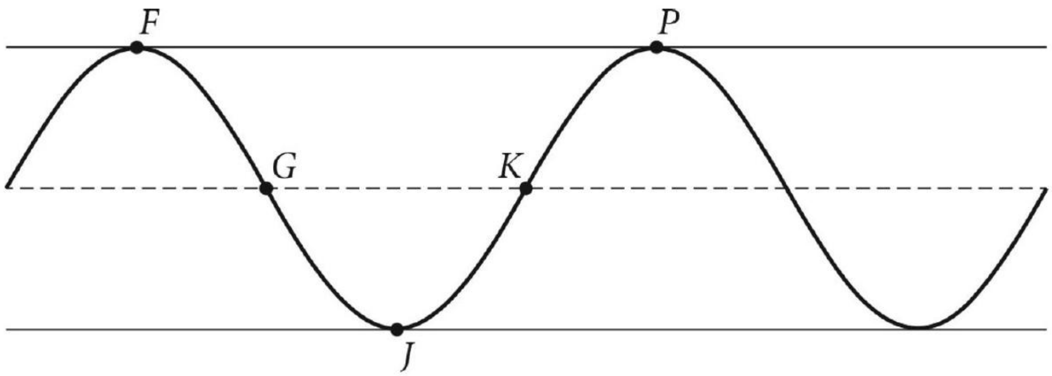
   (B) The function h can be written in the form $h(t) = a\sin\left(b(t + c)\right) + d$. Find values of constants $a, b, c,$ and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of F is t_1 and the t-coordinate of G is t_2.
   (i) On the interval (t_1, t_2), which of the following is true about h?
      a. h is positive and increasing
      b. h is positive and decreasing
      c. h is negative and increasing
      d. h is negative and decreasing
   (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

7. As a human runs, the height of the head increases and decreases periodically. For a certain person's gait pattern, the maximum height of his head is 178 cm and the minimum value of the height of his head is 170 cm. This person takes 3 complete strides per second as he runs. At time $t = 0$, his height is at a minimum.

The sinusoidal function h models the height of this person's head, in centimeters, as a function of time t in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: $F, G, J, K,$ and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants $a, b, c,$ and d.

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1 and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h?
   a. h is positive and increasing
   b. h is positive and decreasing
   c. h is negative and increasing
   d. h is negative and decreasing

(ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

Solutions to #1: Ferris Wheel

(A) F has coordinates $(-7{,}137)$
G has coordinates $(0, 75)$
J has coordinates $(7, 13)$
K has coordinates $(14, 75)$
P has coordinates $(21, 137)$
Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.
Part (A) is worth 2 points total:
1 point for $h(t)$-coordinates
1 point for t-coordinates

(B) $a = 62$
$b = \frac{\pi}{14}$
$c = -21$
$d = 75$
$h(t) = 62\cos\left(\frac{\pi}{14}(t - 21)\right) + 75$
Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.
Part (B) is worth 2 points total:
1 point for vertical transformations: values of a and d
1 point for horizontal transformations: values of b and c

(C)
(i) Choice A
(ii) The graph of h is concave up on the interval (t_1, t_2). The rate of change of h is increasing on the interval (t_1, t_2).
Part (C) is worth 2 points total:
1 point for part (i)
1 point for part (ii)

Solutions to #2: Spit Roast

(A) F has coordinates $(-3.75{,}36)$
G has coordinates $(-2.5, 42)$
J has coordinates $(-1.25, 36)$
K has coordinates $(0, 30)$
P has coordinates $(1.25, 36)$
Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.
Part (A) is worth 2 points total:
1 point for $h(t)$-coordinates
1 point for t-coordinates

(B) $a = 6$
$b = \frac{2\pi}{5}$
$c = 2.5$
$d = 36$
$h(t) = 6\cos\left(\frac{2\pi}{5}(t + 2.5)\right) + 36$
Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.
Other correct forms for $h(t)$ are $h(t) = -6\cos\left(\frac{2\pi}{5}t\right) + 36$ or $h(t) = 6\cos\left(\frac{2\pi}{5}(t - 2.5)\right) + 36$
Part (B) is worth 2 points total:
1 point for vertical transformations: values of a and d
1 point for horizontal transformations: values of b and c

(C)
(i) Choice A
(ii) The graph of h is concave down on the interval (t_1, t_2). The rate of change of h is decreasing on the interval (t_1, t_2).
Part (C) is worth 2 points total:
1 point for part (i)
1 point for part (ii)

Solutions to #3: Paddle Wheel

(A) F has coordinates $(-2,5)$

G has coordinates $(-1,-5)$

J has coordinates $(0,-15)$

K has coordinates $(1,-5)$

P has coordinates $(2,5)$

Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Part (A) is worth 2 points total:

1 point for $h(t)$-coordinates

1 point for t-coordinates

(B) $a = 10$

$b = \frac{\pi}{2}$

$c = 2$

$d = -5$

$h(t) = 10\cos\left(\frac{\pi}{2}(t+2)\right) - 5$

Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.

Other correct answers would be $h(t) = -10\cos\left(\frac{\pi}{2}t\right) - 5$ *or* $h(t) = 10\cos\left(\frac{\pi}{2}(t-2)\right) - 5$.

Part (B) is worth 2 points total:

1 point for vertical transformations: values of a and d

1 point for horizontal transformations: values of b and c

(C)

(i) Choice D

(ii) The graph of h is concave up on the interval (t_1, t_2). The rate of change of h is increasing on the interval (t_1, t_2).

Part (C) is worth 2 points total:

1 point for part (i)

1 point for part (ii)

Solutions to #4: Daylight in Fairbanks

(A) F has coordinates $(0,22)$

G has coordinates $(3,13)$

J has coordinates $(6,4)$

K has coordinates $(9,13)$

P has coordinates $(12,22)$

Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Part (A) is worth 2 points total:

1 point for $h(t)$-coordinates

1 point for t-coordinates

(B) $a = 9$

$b = \frac{\pi}{6}$

$c = -9$

$d = 13$

$h(t) = 9\sin\left(\frac{\pi}{6}(t-9)\right) + 13$

Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.

Another correct answer is $h(t) = -9\sin\left(\frac{\pi}{3}(t-3)\right) + 13$. *A third correct answer is* $9\sin\left(\frac{\pi}{3}(t+3)\right) + 13$

Part (B) is worth 2 points total:

1 point for vertical transformations: values of a and d

1 point for horizontal transformations: values of b and c

(C)

(i) Choice B

(ii) The graph of h is concave up on the interval (t_1, t_2). The rate of change of h is increasing on the interval (t_1, t_2).

Part (C) is worth 2 points total:

1 point for part (i)

1 point for part (ii)

Solutions to #5: Washing Machine

(A) F has coordinates $(0,52.5)$

G has coordinates $\left(\frac{1}{80}, 75\right)$

J has coordinates $\left(\frac{1}{40}, 52.5\right)$

K has coordinates $\left(\frac{3}{80}, 30\right)$

P has coordinates $\left(\frac{1}{20}, 52.5\right)$

Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Part (A) is worth 2 points total:

1 point for $h(t)$-coordinates

1 point for t-coordinates

(B) $a = 22.5$

$b = 40\pi$

$c = 0$

$d = 52.5$

$h(t) = 22.5\sin(40\pi t) + 52.5$

Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.

Part (B) is worth 2 points total:

1 point for vertical transformations: values of a and d

1 point for horizontal transformations: values of b and c

(C)

(i) Choice B

(ii) The graph of h is concave down on the interval (t_1, t_2). The rate of change of h is decreasing on the interval (t_1, t_2).

Part (C) is worth 2 points total:

1 point for part (i)

1 point for part (ii)

Solutions to #6: Helicopter Tail Rotor

(A) F has coordinates $(0,17)$

G has coordinates $(3, 12)$

J has coordinates $(6, 7)$

K has coordinates $(9, 12)$

P has coordinates $(12, 17)$

Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Part (A) is worth 2 points total:

1 point for $h(t)$-coordinates

1 point for t-coordinates

(B) $a = 5$

$b = \frac{\pi}{6}$

$c = -9$

$d = 12$

$h(t) = 5\sin\left(\frac{\pi}{6}(t-9)\right) + 12$

Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.

Another correct form of $h(t)$ is $h(t) = -5\sin\left(\frac{\pi}{6}(t-3)\right) + 12$

Part (B) is worth 2 points total:

1 point for vertical transformations: values of a and d

1 point for horizontal transformations: values of b and c

(C)

(i) Choice B

(ii) The graph of h is concave down on the interval (t_1, t_2). The rate of change of h is decreasing on the interval (t_1, t_2).

Part (C) is worth 2 points total:

1 point for part (i)

1 point for part (ii)

Solutions to #7: Human Running Head Height

(A) F has coordinates $\left(-\frac{1}{6}, 178\right)$

G has coordinates $\left(-\frac{1}{12}, 174\right)$

J has coordinates $(0, 170)$

K has coordinates $\left(\frac{1}{12}, 174\right)$

P has coordinates $\left(\frac{1}{6}, 178\right)$

Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Part (A) is worth 2 points total:

1 point for $h(t)$-coordinates

1 point for t-coordinates

(B) $a = 4$

$b = 6\pi$

$c = -\frac{1}{12}$

$d = 174$

$h(t) = 4\sin\left(6\pi\left(t - \frac{1}{12}\right)\right) + 174$

Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.

Part (B) is worth 2 points total:

1 point for vertical transformations: values of a and d

1 point for horizontal transformations: values of b and c

(C)

(i) Choice A

(ii) The graph of h is concave down on the interval (t_1, t_2). The rate of change of h is decreasing on the interval (t_1, t_2).

Part (C) is worth 2 points total:

1 point for part (i)

1 point for part (ii)